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Shevtsova M.V.[1]
Averin G.V.[2]

QUASILINEAR DIFFERENTIAL EQUATIONS FOR THE DESCRIPTION OF THE SPACE OF IDEAL GAS CONDITIONS

[1] Belgorod State Technological University named after V.G. Shoukhov,
46 Kostyukova St, Belgorod, 308012, Russia

[2] Belgorod State University, 85 Pobeda St, Belgorod, 308015, Russia

e-mail: averin@bsu.edu.ru, mashashev81@gmail.com

Abstract

Relevance

Today the classical thermodynamics as fundamentals of many physical sciences does not possess the finished and accurate axiomatic creation of the theory. Its many provisions and ratios are based on the empirical facts which are recognized as apriori and are not proved in terms of theoretical parcels.

Problem

In this paper the problem of a wording of thermodynamic provisions and ratios for spaces of ideal gas conditions is considered on the basis of analysis of solutions of partial differential equations of the first order.

Methods

In this work the method of characteristics for the solution of the quasilinear differential equations of the first order was used. And also formulas and dependences of differential geometry and means of computer mathematics are applied.

Results

It is shown that characteristics of partial differential equations are connected with entropy as a thermodynamic function of condition. Geometric presentation of the received integrated surfaces is executed. The connection between physical content of thermodynamic sizes (temperature, entropy, energy) and their mathematical analogs is established. By numerical methods using the means of computer mathematics it is illustrated the possibility of establishing consistent patterns of implementation of thermodynamic processes and cycles at the description them as functions of time.

Conclutions

The assumption is formulated that irreversibility of thermodynamic processes can be connected with temporal features of implementation of these processes. The offered approach allows to give simple geometric interpretation of basic provisions and ratios of classical thermodynamics.

Keywords: ideal gas; provisions and ratios of thermodynamics; geometric interpretation.

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Шевцова М.В.[1]
Аверин Г.В.[2]

КВАЗИЛИНЕЙНЫЕ ДИФФЕРЕНЦИАЛЬНЫЕ УРАВНЕНИЯ ПЕРВОГО ПОРЯДКА ДЛЯ ОПИСАНИЯ СОСТОЯНИЯ ИДЕАЛЬНОГО ГАЗА

[1] Белгородский государственный технологический университет им. В.Г. Шухова,
ул. Костюкова, 46, г. Белгород, 308012, Россия

[2] Белгородский государственный национальный исследовательский университет,
ул. Победы, 85, г. Белгород, 308015, Россия

e-mail: averin@bsu.edu.ru, mashashev81@gmail.com

Аннотация

Актуальность

Классическая термодинамика как основа многих физических наук на настоящее время не обладает законченным и четким аксиоматическим построением теории. Некоторые из ее положений и соотношений базируются на эмпирических фактах, признаются априорными и не обоснованы с точки зрения теоретических посылок.

Проблема
В данной статье рассматривается задача формулировки положений и соотношений термодинамики для пространства состояний идеального газа на основе анализа решений дифференциальных уравнений в частных производных первого порядка.
Методы
В настоящей работе были использованы метод характеристик для решения квазилинейных дифференциальных уравнений первого порядка, а также формулы и зависимости дифференциальной геометрии и средства компьютерной математики.
Результаты
Показана связь характеристик дифференциальных уравнений в частных производных с энтропией, как термодинамической функцией состояния. Выполнено геометрическое представление полученных интегральных поверхностей и установлена взаимозависимость между физическим содержанием термодинамических величин (температуры, энтропии, энергии) и их математическими аналогами. Показано, что численными методами с использованием средств компьютерной математики можно установить закономерности осуществления термодинамических процессов и циклов при описании их функциями времени.
Выводы
Сформулировано предположение, что необратимость термодинамических процессов может быть связана с темпоральными особенностями осуществления этих процессов. Предложенный подход позволяет дать простую геометрическую интерпретацию основным положениям и соотношениям классической термодинамики.

Ключевые слова: идеальный газ; положения и соотношения термодинамики; геометрическая интерпретация.

INTRODUCTION

The classical thermodynamics is a theoretical basis for many physical sciences. But thermodynamics’ theory is not full, its many aspects are contradictory and tangled, a number of provisions has no logical clarity [3, 11]. Axiomatic creation of thermodynamics, despite many works in this area [2, 4, 5, 7, 10, 13, 17–20], is not completed. The problem of entropy and development of various systems of justification of its existence is connected with thermodynamic axiomatics. The fact of existence of entropy as one of basic provisions in thermodynamics, the fundamental principle of its increase and communication of entropy with irreversibility of processes in the nature are not completely studied.

The second problem is related with the fact that operating with thermodynamic processes which proceed in time, the classical thermodynamics does not give the answer to a question what is the place of time in the theory. Time exception paradox in ratios of classical thermodynamics is connected with a concept of equilibrium (infinitely slow) process. However, the set of experimental justifications in thermodynamics is not connected at all with implementation very slow (equilibrium, quasistatic) processes [12].

It is impossible to claim that the entity of a problem of irreversibility is determined by a slow or rapid current of process. However, the hypothesis that irreversibility is related with features and patterns of development of thermodynamic processes in time has the right for existence. Therefore, this problem can be studied only at entering of time parameter to the equations of classical thermodynamics.

To understand the issues stated above, let’s address to a concept of ideal gas. This model is rather simple and evident and allows geometric interpretation of the basic concepts and ratios in three-dimensional space. However, the analysis of references is showed that the works devoted to disclosure of geometric sense and interpretation of thermodynamic sizes, ratios and laws are not enough. This direction of researches is affected most of all in K. Caratheodory [7], Falk and Jung [20] and Mlodzeevsky's works [9]. There is no fair and evident idea about connection between thermodynamic processes and objects and geometrical structures and the relations yet. Proceeding from this, there are no obvious analogies between the physical maintenance of thermodynamic values (temperature, heat, entropy, energy, etc.) and their mathematical models.

In works of authors [1, 14, 15, 16] attempts of application of methods and means of differential geometry at justification of models of thermodynamic conditions, processes, ratios and regularities were made earlier. In our opinion, perspective approach in the solution of this task lies in the analysis of solutions of partial differential equations of the first order and differential Pfaff forms [6] which are the basis for the theory of thermodynamics.

So, the purpose of this article is establishment of communication between the physical maintenance of the main thermodynamic values and their models on the basis of application of methods and means of differential geometry. It will allow to give geometric interpretation to basic provisions and ratios of classical thermodynamics. The research of this task at the description of processes in time is possible both by the analytical solution of the differential equations of thermodynamics, and the application of methods and means of computer mathematics.

MAIN PART

PURPOSE OF WORK

The thermodynamic system usually refers to a set of macroscopic bodies and fields of the physical nature, which represents a complete object and interact as among themselves and with the environment. At the same time the condition of a system is defined as a set of its thermodynamic properties which parameters are formed under the influence of environmental conditions in particular timepoint. It means for ideal gas that each its state is unambiguously defined by values of specific volume v and pressure p. Let's assume that at commission in time of any process l parameters of a condition of ideal gas can be presented by the parametrical equations concerning time τ: $\mathrm{v} = \mathrm{v}(\tau)$ and $p = p(\tau)$.

Let's use the skilled fact of temperature existence. Temperature T is called the measure of a deviation of a condition of the studied thermodynamic system from a condition of thermal balance of a reference body in the standardized conditions. The corresponding reference body is called the thermometer. Depending on what reference body is accepted as the thermometer, different scales of empirical temperatures are distinguished. At the same time the ideal gas scale represents a special form of a measuring temperature scale.

The next skilled fact is existence of a concept of amount of heat and thermal capacities. Amount of heat Q is the physical quantity characterizing process of heat exchange between a thermodynamic system and the environment. Thermal capacity c_l is entered in physics as a special type of value which is one of thermal characteristics of substance. There is a set of methods of determination of thermal capacities of gases, solids and liquids in experience [12]. The equation determining the amount of heat in process which is necessary for temperature change of a body is usually presented concerning temperature and thermal capacity of a body in the form:

$$\mathrm{c_l} = \left(\frac{\mathrm{dQ}}{\mathrm{qT}}\right)_{\mathrm{l}} \tag{1}$$

Thus, proceeding from the experimental data, it is possible to claim that the amount of heat and temperature are connected with pressure and specific volume of ideal gas.

Let's construct on the plane of Cartesian coordinates the geometric system in the form of the space of condition of ideal gas where coordinate axes correspond to independent variables v and p. As between values Q and T in any process there is a communication (1), ratios will be correct:

$$\frac{\partial Q}{\partial v} = c_p \frac{\partial T}{\partial v} \text{ and } \frac{\partial Q}{\partial p} = c_v \frac{\partial T}{\partial p}, \tag{2}$$

where c_p and c_v are thermal capacity of ideal gas with a constant pressure and constant volume respectively.

According to Klapeyron's equation $T = pv/R_i$ temperature has a look of uniform function of the second degree. So it satisfies to Euler's formula:

$$T = \frac{1}{2}\left(v\frac{\partial T}{\partial v} + p\frac{\partial T}{\partial p}\right) \tag{3}$$

Taking into account ratios (1) this equation can be presented in the form of the linear non-uniform partial equation of the first order concerning value Q:

$$\frac{v}{2c_p}\frac{\partial Q}{\partial v}+\frac{p}{2c_v}\frac{\partial Q}{\partial p}=T \tag{4}$$

The solution $Q=Q(v,p)$ for the equation (4) geometrically represents a surface in three-dimensional space (v,p,Q) which is called an integrated surface. Let’s use method of characteristics which are defined by the system of the ordinary differential equations [8, p. 41]:

$$2c_p\frac{dv}{v}=2c_p\frac{dp}{p}=\frac{dQ}{T}=ds, \tag{5}$$

where s is any real parameter. If we will determine parameter s as the arch length changing along a characteristic curve, the equations (5) will take a form:

$$\frac{dv}{ds}=\frac{v}{2c_p};\ \frac{dp}{ds}=\frac{p}{2c_v};\frac{dQ}{ds}=\frac{pv}{R_i}=T \tag{6}$$

From the first two equations (5) the dependence for the value ds turns out. It has a form of known thermodynamic equation used in definition of entropy of ideal gas:

$$ds=\frac{dQ}{T}=2c_p\frac{dv}{v}+2c_v\frac{dp}{p};$$

$$s-s_0=c_p ln\frac{v}{v_0}+c_v ln\frac{p}{p_0} \tag{7}$$

Thus, in geometric representation the entropy is arch length for the characteristic curves corresponding to the field of the directions which is defined by the system of the equations (5). From the theory it is also known [8, p.42] that the integrated solution $Q=Q(v,p)$ of the equation (4) can be covered with collection of characteristics. And any characteristic curve determined by the equations (5) and having the common point with the integrated surface entirely lies on this surface.

The integrated solution of the equation (4) can be found in an analytical way. Cauchy problem for this equation is connected with finding of the integrated surface $Q=Q(v,p)$ passing through the set curve of any process *l* which can be presented in a parametrical form concerning time τ:

$$v_l=\ v_l(\tau);p_l=\ p_l(\tau);Q_l=\ Q_l(\tau).$$

The general solution of a system of the equations (5) concerning entropy has a look:

$$v=\ v_l exp\left(\frac{s}{2c_p}\right);p=\ p_l exp\left(\frac{s}{2c_v}\right);$$

$$Q=\ Q_l+c_p\beta_1\frac{p_l v_l}{R_i}\left(exp\left(\frac{s}{c_p\beta_1}\right)-1\right);$$

$$\beta_1=\frac{2c_v}{c_p+c_v}. \tag{8}$$

For receiving the integrated surface in three-dimensional space (v,p,Q) we set parametrically a curve of process *l* and exclude values s and τ [8, p. 42].

Let's assume that process of *l* is isobaric, and the line of process *l*, through which the integrated surface has to pass, is set by the parametrical equations concerning time τ:

$$v_l=\ v_l+\alpha_v\tau;p_l=p_1;$$

$$Q=c_pT_l=c_p\frac{(v_l+\alpha_v\tau)p_1}{R_i} \tag{9}$$

v_l, p_l - thermodynamic parameters of gas in some initial point where τ=0.

Having substituted (9) in (8) and excluding values s and τ we will receive:

$$Q=c_p\frac{pv}{R_i}\left(\beta_1+\beta_2\left(\frac{p}{p_1}\right)^{-\frac{k+1}{k}}\right), \tag{10}$$

$k=c_p/c_v$ - adiabatic curve indicator.

Similar results can be received by numerical methods, using means of computational mathematics. Let that condition of hydrogen in initial timepoint has parameters –pressure $p_1 = 101325$ Pa and specific volume $v_1 = 11{,}12720$ m^3/kg.

At expansion in isobaric thermodynamic process during 100 sec. specific volume increased to $v_2 = 15{,}20087$ m^3/kg. At the same time temperature have changed from $T_1 = 273{,}15$ K to $T_2 = 373{,}15$ K.

We construct the integrated surface which is the solution of the equation (4), using the system of computer mathematics Maple (Fig. 1).

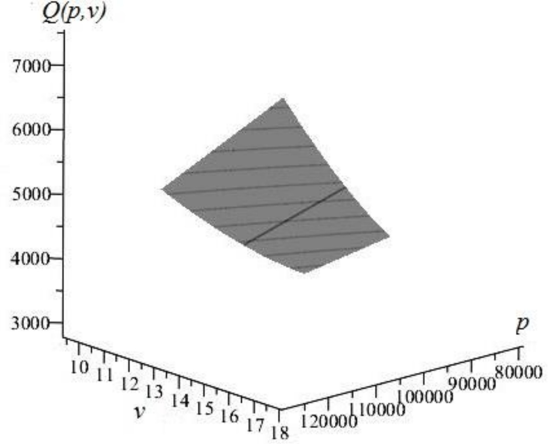


Рис. 1. Интегральная поверхность для количества теплоты при изобарном процессе расширения водорода
Fig 1. Integrated surface for amount of heat at isobaric process of expansion of hydrogen

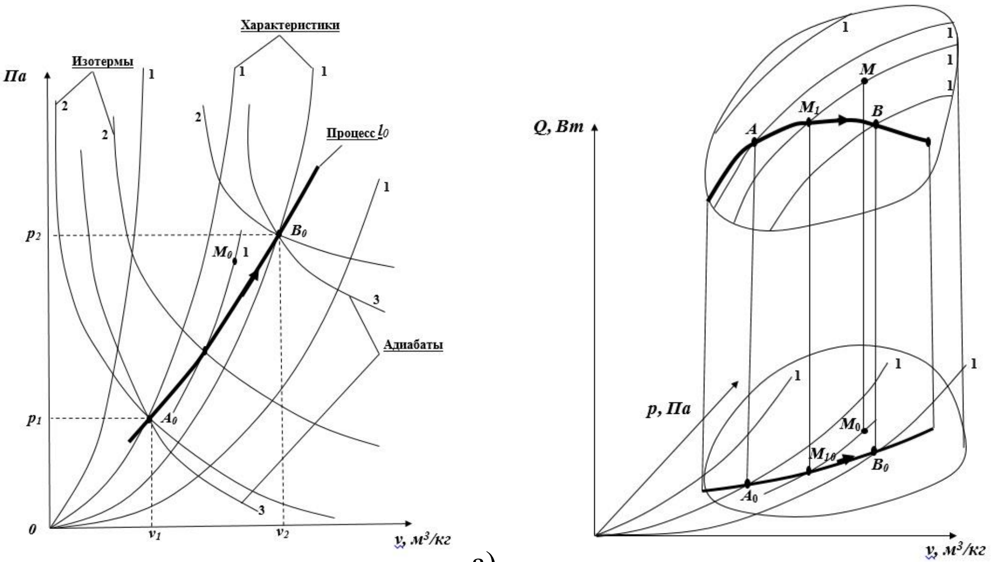
Thus, on the basis of solutions of partial differential equations of the first order it is possible to describe changes in time of conditions of ideal gas at thermodynamic processes.

GEOMETRIC INTERPRETATION OF THERMODYNAMIC VALUES

It is known that integrated surfaces of the quasilinear equation (4) can be covered with the collection of characteristics which are defined by equations (5). The functions $f_1 = \frac{v}{2c_p}, f_2 = \frac{p}{2c_v}, f_3 = T$ in the equation (4) define a field of the directions in space (v, p, Q) where in each point of this space there is a direction which directional cosines are proportional to f_1, f_2, f_3. Thus, the equation (4) resolves itself into requirement that in each point of integrated surface $Q = Q(v, p)$ vector determined by the field of the directions stated above have to be in the tangent plane to this surface [8, p. 41].

Let process l in space (v, p, Q) is set in parametrical form (8). Let's put l_0 a projection of curve l on the vOp plane. Then for the equation (4) Cauchy problem is formulated in a form: to find the integrated surface of the equation (4) passing through the set curve l in the neighborhood of projection l_0. Geometric interpretation of Cauchy problem assumes that in space (v, p, Q) through each point of process l it is necessary to carry out characteristic of the equation (4) and "to stick together" from them an integrated surface.

On the vOp plane it looks as follows (fig. 2, a). There is a projection of process l_0 from the beginning at point $A_0(v_1, p_1)$ and the end at point $B_0(v_2, p_2)$. Through points A_0 and B_0 characteristics 1, isotherms 2 and adiabatic curves 3 pass. The collection of characteristics is described by the equation $p = Cv^k$, proceeding from the first integral of a system (5), collection of isotherms – $pv = C$. In turn, the collection of adiabatic curves is described by the equation $pv^k = C$. The entropy (7) in such representation will be characteristic's arc length, and adiabatic curves will represent lines of level for characteristics at *s=const*. Adiabatic curves, isotherms and characteristics in pairs form some network of curvilinear not orthogonal coordinates on the vOp plane.



a) b)

Рис. 2. Геометрическая интерпретация задачи Коши для уравнения (4):
а – на плоскости vOp; б – в пространстве (v, p, Q)
Fig. 2. Geometric interpretation of Cauchy's problem for equation (4):
a – on vOp plane; б – in (v, p, Q) space

The geometric solution of Cauchy problem in (v, p, Q) space is constructed as follows. Through any point M$_0$ on vOp plane characteristic is carried out until its crossing with process *l$_0$*. After that it is necessary to put $Q = Q_l(\tau)$ taking into account the parametrical equations of process l (fig. 2, b). The integrated surface $Q = Q(v, p)$ will characterize amount of heat for all set of conditions of ideal gas in the neighborhood of process l or its projection *l$_0$*.

In space (v, p, Q) it is possible to construct a surface of energy $u = u(v, p)$ for conditions of ideal gas, using for this purpose the equation $u = c_p T = c_p \frac{pv}{R_i}$. Then for any process *dl* change of amount of heat can be presented in a differential form like the sum of two functions $dQ = du + F(p, v)$. This ratio will have an appearance of energy conservation equation. At the same time a problem of definition of function $F(p, v)$ is arised which can be solved by means of computational mathematics. It will allow to establish provisions under which the differential form $dQ = du + F(p, v)$ can have the known appearance of energy conservation equation $dQ = du + pdv$.

CONCLUSIONS

The offered approach gives the chance to carry out geometric interpretation of basic provisions and ratios of thermodynamics as Cauchy problem for partial quasilinear differential equations of the first order has the evident geometric image in multidimensional spaces of conditions. It allows to give a mathematical definition for entropy and to present it as an arc length of characteristic of the equation (4). In turn, time and entropy describe states and processes of change of conditions of ideal gas in parametrical form. Presentation of these physical values as parameters gives a chance to enter time into the equations of classical thermodynamics and to give simple geometric interpretation to its basic provisions.

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Шевцова Мария Витальевна, доцент кафедры программного обеспечения вычислительной техники и автоматизированных систем, кандидат физико-математических наук
Аверин Геннадий Викторович, заведующий кафедрой общей математики, доктор технических наук, профессор

Shevtsova Maria Vitalevna, Associate Professor of Computer Software and Automated Systems, Candidate of Physical and Mathematical Sciences
Averin Gennady Viktorovich, Head of the Department of General Mathematics, Doctor of Technical Sciences, Professor